of the phase equilibrium.

In conclusion the author thanks L. A. Galin, S. I. Anisimov and M. Ia. Azbel' for assessment of this paper.

BIBLIOGRAPHY

1. Molchanov, A. M., Boundedness of variation of continuous solutions of the system of hydrodynamic equations. Dokl. Akad. Nauk SSSR, Vol. 129, №6, 1959.
2. Rozhdestvenskii, B. L. and Ianenko, N. N., System of Quasilinear Equations. M., "Nauka", 1968.
3. Tsurkov, V. I., Gasdynamic effects related to the boundedness of solutions of quasilinear systems. J. USSR Comput. Math. and Math. Phys., Vol. 11, №2, Pergamon Press, 1971.
4. Azbel', M. Ia., Voronel', A. V. anf Giterman, M. Sh., On the theory of critical point. ZhEFT, Vol. 46, №2, 1964.
5. Tsurkov, V. I., On a selfsimilar solution of the gasdynamic equations. J. USSR Comput. Math. and Math. Phys., Vol. 11, №4, Pergamon Press, 1971.
6. Landau, L. D. and Lifshits, E. M., Theoretical Physics, Vol. 5, Statistical Physics (English translation), Pergamon Press, 1968.

Translated by L. K.

UDC 534.222.2

EXPLOSIONS IN DETONATING MEDIA OF VARIABLE INITIAL DENSITY

PMM Vol. 37, №1, 1973, pp. 172-177
N. S. MEL'NIKOVA
(Moscow)
(Received May 6, 1971)

We consider a non-self-similar problem of point explosion in a detonating gas, in a medium of variable initial density. Analytic expressions are obtained showing the dependence of the pressure, density and gas velocity on the distance from the origin of explosion and the radius of detonation wave, the latter obtained by solving a differential equation. Computations are performed for the cases of spherical and cylindrical symmetry for various values of the adiabatic exponent, and the variation of initial density exponent.

Let us consider a perfect gas which is inviscid and non-heat-conducting. Suppose that an instantaneous explosion of finite energy E_0 occurs at the instant $t = 0$ in an unbounded medium at rest $(v_1 = 0)$ at a point, or along a plane, or along a staight line [1]. The explosion generates a strong shock wave which propagates through the gas and heats it up to the state at which rapid combustion becomes possible. Assuming that the energy E_0 is large and much larger than the amount of energy Q_1 released during the gas combustion, we can infer that the gas burns in the direct vicinity of the shock-wave front. In this case we can consider the shock wave and the chemical reaction zone together, as a single surface of a strong explosion with release of heat, i.e. treat it as a detonation-wave front.

Let us denote by p_1 the initial pressure and by ρ_1 the initial density depending on the initial coordinate ξ of the particle in accordance with the law $\rho_1 = A\xi^{-\omega}$, where A is a positive dimensional constant and ω is an abstract constant which may be positive or negative. The case $\omega = 0$ corresponds to the constant initial density.

Let Q be the heat generating capacity of a unit mass of the burning gas. Then the energy Q_1 released during the combustion process by the time the radius of the wavefront becomes equal to r_2, is

$$Q_1 = \sigma_\nu(\nu - \omega)^{-1} \rho_1 Q r_2^\nu, \quad \sigma_\nu = 2 (\nu - 1)\pi + (\nu - 2) (\nu - 3)$$

where $\nu = 1$, $\nu = 2$ and $\nu = 3$ for the plane, cylindrical and spherical waves, respectively.

From the physical considerations it follows that during the first instances following the explosion the gas will move according to the laws governing a point explosion without detonation, as the contribution of the energy of combustion to the total energy content will be small. Assuming that the energy of explosion E_0 is much larger than the energy of combustion Q_1, we can find a region of flow $r_2 < r_2^*$, $r_2^* = (\nu E_0 / \sigma_\nu AQ)^{1/\nu+\omega}$ in which the detonation exerts only a weak influence [2, 3].

On increasing the radius of the detonation wave the energy of combustion Q_1 increases as well. For this reason the influence of combustion must be taken into account in the condition of conservation of energy, at the same time retaining unchanged the mechanical conditions at the wave.

Below we propose an approximate method of solution and consider a non-self-similar problem of explosion in which the energy of combustion Q_1 at the wavefront is taken into account. The linearized formulation of this problem was considered in [4].

The solution of the problem under consideration is reduced to integrating a system of equations of gasdynamics for one-dimensional motions, with the initial conditions at the center and the boundary conditions at the detonation-wave front both taken into account. The system of equations of gasdynamics describing one-dimensional adiabatic motions of the gas during explosion is taken in the following form:

$$\begin{gathered} \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \\ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial r} + \frac{(\nu - 1)\rho v}{r} = 0 \\ \frac{d\varepsilon}{dt} - \frac{p}{\rho^2}\frac{d\rho}{dt} = 0 \end{gathered} \tag{1}$$

At the instant $t = 0$ a finite amount of energy E_0 is released at the center of symmetry and the following initial conditions are specified:

$$\begin{gathered} v(r, 0) = 0, \quad \rho(r, 0) = \rho_1 = A\xi^{-\omega} \\ p(r, 0) = p_1 = \text{const}, \quad r_2(0) = 0 \end{gathered} \tag{2}$$

The following laws of conservation of mass, energy and the rate of change of momentum must hold at the detonation-wave front

$$\begin{gathered} \rho_1 (v_1 - c) = \rho_2 (v_2 - c), \ \ \rho_1 (v_1 - c)^2 + p_1 = \rho_2 (v_2 - c)^2 + p_2 \\ \frac{1}{2}(v_1 - c)^2 + \frac{\gamma_1 p_1}{(\gamma_1 - 1)\rho_1} + Q = \frac{1}{2}(v_2 - c)^2 + \frac{\gamma}{(\gamma - 1)}\frac{p_2}{\rho_2} \end{gathered} \tag{3}$$

where c is the shock wave velocity and γ_1 and γ are specific heat capacities ahead and behind the wavefront.

Thus, in order to solve the problem of point explosion in a detonating gas, we must find the solution of the system (1) satisfying the boundary conditions (2) and (3). The system of defining parameters for the perturbed motion following an explosion, consists of the following quantities

$$E_0,\ Q,\ A,\ p_1,\ \omega,\ \gamma,\ r,\ t,\ \gamma_1 \tag{4}$$

For simplicity we shall assume the detonation wave to be strong, in which case we can ignore the initial pressure p_1 and then γ_1 will vanish from the conditions (3) at the wavefront. In the case of a strong detonation wave the system (4) of defining parameters implies that, when the dimensionless unknown functions and variables

$$f(\lambda, q) = v/v_2, \quad g(\lambda, q) = \rho/\rho_2, \quad h(\lambda, q) = p/p_2 \tag{5}$$

$$\lambda = r/r_2, \ q = \sqrt{Q/c}$$

are introduced, then the dimensionless functions $f(\lambda, q)$, $g(\lambda, q)$ and $h(\lambda, q)$ will also depend on the constant parameters γ and ω. The conditions at the strong detonation wave $(p_1 = 0)$ can be written for a gas at rest $(v_1 = 0)$ in the form

$$v_2 = c(1-\varepsilon), \quad p_2 = \rho_1 c^2 (1-\varepsilon), \ \rho_2 = \rho_1/\varepsilon$$

$$\varepsilon = (\gamma+1)^{-1}[\gamma - \sqrt{1 - 2(\gamma^2-1) q^2}] \tag{6}$$

Using the dimensionless variables (5) we can write the conditions at the detonation wave in the form

$$f_2(1, q) = 1, \ g_2(1, q) = 1, \ h_2(1, q) = 1 \tag{7}$$

When Q is arbitrary, the parameter q becomes variable and the problem will consequently be non-self-similar. At small values of q the problem can be solved by linearizing the initial equations (1) with respect to small parameter q about the self-similar solution [5, 8].

In the present paper the non-self-similar problem is solved by specifying one of the functions characterizing the motions by means of an interpolation formula. The time-dependent coefficients of this formula are obtained using the integral laws of conservation and the boundary conditions (6). The remaining functions characterizing the motions are found from the exact equations of motion [9, 10].

The characteristics $v_2(t)$, $\rho_2(t)$ and $p_2(t)$ at the detonation wave can be found from the relation (6), provided that $r_2(t)$ is known. To find $r_2(t)$ we shall utilize the law of conservation of energy connecting the energy of explosion E_0 and the energy of combustion Q_1 with the parameters of the detonation-wave front.

If $p_1 = 0$, then the integral law of conservation of energy can be written in the dimensionless variables (5) in the form

$$j_1(\varepsilon) I_1 + j_2(\varepsilon) I_2 = \frac{q^2}{\nu-\omega} + \frac{q^2}{\sigma_\nu \gamma R^{\nu-\omega}} \tag{8}$$

where

$$j_1(\varepsilon) = \frac{p_2}{(\gamma-1)\rho_1 c^2} = \frac{(1-\varepsilon)}{\gamma-1}, \quad j_2(\varepsilon) = \frac{\rho_2 v_2^2}{2\rho_1 c^2} = \frac{(1-\varepsilon)^2}{2\varepsilon}$$

$$I_1 = \int_0^1 h\lambda^{\nu-1} d\lambda, \qquad I_2 = \int_0^1 g f^2 \lambda^{\nu-1} d\lambda$$

Let us introduce the dimensionless quantities R and τ

$$R = \frac{r}{r_0}, \quad \tau = \frac{t}{t_0}, \quad r_0 = \left(\frac{\gamma E_0}{QA}\right)^{1/\nu-\omega}, \quad t_0 = \frac{r_0}{\sqrt{Q}}$$

where r_0 denotes the dynamic length at t_0 the dynamic time.

The integral law can be used with advantage to determine the radius $r_2(t)$ of the shock wave, provided the solution of (1) has been found. If the dependence of the Eulerian coordinate r on t and on the Lagrangian coordinate ξ is known, then the system (1) yields analytic expressions for the velocity, density and pressure in terms of the radius r and the radius of the detonation wave. We shall seek the function r in the following form:

$$r = c(t)\,\xi^{\alpha_1} + b \tag{9}$$

When $c(t)$, $\alpha_1(t)$ and b are chosen properly, the system (1) and the corresponding boundary conditions (2) and (3) can be satisfied. We use the initial coordinates ξ of the particle as the Lagrangian coordinate, the latter assuming the value $\xi = r_2$ at the instant at which the shock wave passes the particle. Then the coefficients appearing in (9) become

$$c(t) = r_2^{(1-\alpha_1)}, \qquad b = 0 \tag{10}$$

The differential form of the law of conservation of mass together with (9) and (10) yield

$$\rho = \rho_2\left(\frac{r}{r_2}\right)^{\alpha(t)} \tag{11}$$

$$\alpha(t) = \nu\left(\frac{\rho_2}{\rho_1} - 1\right) - \omega\frac{\rho_2}{\rho_1}, \qquad \alpha_1(t) = \frac{\rho_1}{\rho_2}$$

In the following we shall assume, when solving the problem, that the density distribution within the shock wave is given by the formulas (11). Then the second equation of (1) gives the velocity of the gas, and the first equation its pressure. The third equation of (1), which is the energy equation, can be used to determine the radius $r_2(t)$ of the shock wave, although the integral law of conservation of energy (8) is more convenient for this purpose. For this reason, from now on we shall use the latter in the region of perturbed motion contained within the shock wave.

Inserting the expression for $\rho(r, t)$ from (11) into the second equation of (1) we obtain an equation for the velocity, which, when solved with the boundary conditions (2) and (3) taken into account, yields

$$v(r, t) = v_2\left(1 - \frac{r_2}{v_2}\ln\frac{r}{r_2}\frac{d}{dt}\ln\frac{\rho_2}{\rho_1}\right)\frac{r}{r_2} \tag{12}$$

Inserting $v(r, t)$ from (12) and $\rho(r, t)$ from (11) into the first equation of (1) we obtain an equation for the pressure. The latter, with the boundary condition (3) taken into account, can be solved to give the pressure.

After some simplification we obtain the formulas for the distribution of the dimensionless characteristics of the motion in the perturbed region in the following form:

$$h = \frac{p}{p_2} = 1 + \frac{H_1\rho_2 r_2}{(\alpha+2)\,p_2}\left[1 - \left(\frac{r}{r_2}\right)^{\alpha+2}\right] - \frac{\rho_2 r_2}{p_2(\alpha+2)}\left(\frac{r}{r_2}\right)^{\alpha+2}\left(H_2 + H_3\ln\frac{r}{r_2}\right)\ln\frac{r}{r_2}$$

$$f = \frac{v}{v_2} = \left[1 - H_4\ln\frac{r}{r_2}\right]\frac{r}{r_2}, \qquad g = \frac{\rho}{\rho_2} = \left(\frac{r}{r_2}\right)^{\alpha} \tag{13}$$

where

$$H_1 = K_1 - \frac{H_2}{\alpha+2}, \qquad H_2 = K_2 - \frac{2H_3}{\alpha+2}$$

$$H_3 = r_2\left[\frac{d}{dt}\left(\ln\frac{\rho_2}{\rho_1}\right)\right]^2, \qquad H_4 = \frac{r_2}{v_2}\frac{d}{dt}\left(\ln\frac{\rho_2}{\rho_1}\right)$$

$$K_1 = \frac{v_2^2}{r_2} + \frac{dv_2}{dt} - v_2 \frac{d}{dt}\left(\ln \frac{\rho_2}{\rho_1}\right) - \frac{v_2}{r_2}\frac{dr_2}{dt} + \frac{dr_2}{dt}\frac{d}{dt}\left(\ln \frac{\rho_2}{\rho_1}\right)$$

$$K_2 = 2H_3 - 2v_2 \frac{d}{dt}\left(\ln \frac{\rho_2}{\rho_1}\right) - \frac{r_2\rho_1}{\rho_2}\frac{d^2}{dt^2}\left(\frac{\rho_2}{\rho_1}\right)$$

The solution (13) satisfies all boundary conditions. The time t enters this solution through the characteristics of the motion at the detonation wave front and the front coordinate $r_2(t)$. The conditions (6) at the detonation wave show clearly that all characteristics $v_2(t)$, $\rho_2(t)$ and $p_2(t)$ of the motion in the perturbed region are expressed in terms of the function $r_2(t)$ defining the law of propagation of the detonation wave. If the value of $r_2(t)$ is found, e. g. from an experiment, then the formulas (13) give a complete solution of the problem in question, i. e. that concerning an explosion in a medium of variable initial density with combustion taking place at the wavefront.

It must be noted that the formulas (13) are reduced to the same form as those obtained in [7] while solving the problem of point explosion in a medium of variable initial density without combustion taking place at the front, but the dependence of the functions H_1, H_2, H_3 and H_4 on $R_2(q)$ is given by the formulas

$$H_1 = \frac{Q}{r_0}\left\{\left[\frac{\varepsilon-1}{q} - \frac{4(\gamma-1)q}{[\gamma(1-\varepsilon)-\varepsilon]}\right]\frac{1}{q^2}\frac{dq}{dR_2} + \frac{\varepsilon(\varepsilon-1)}{q^2R_2}\right\} - \frac{\varepsilon H_2}{[\nu-\omega+(2-\nu)\varepsilon]}$$

$$\begin{gathered} H_2 = \frac{Q}{r_0}\frac{2(\gamma-1)R_2}{\varepsilon q[\gamma(1-\varepsilon)-\varepsilon]}\left\{\frac{d^2q}{dR_2^2} + 2(1-\varepsilon)\frac{dq}{dR_2} + \right. \\ \left.\left[\frac{1}{[\gamma(1-\varepsilon)-\varepsilon]^2 q} - \frac{4(\gamma-1)q}{[\gamma(1-\varepsilon)-\varepsilon][\nu-\omega+(2-\nu)\varepsilon]} - \frac{1}{q}\right]\left(\frac{dq}{dR_2}\right)^2\right\} \\ H_3 = \frac{Q}{r_0}\frac{4(\gamma-1)^2R_2}{\varepsilon^2[\gamma(1-\varepsilon)-\varepsilon]^2}\left(\frac{dq}{dR_2}\right)^2 \\ H_4 = \frac{2(\gamma-1)qR_2}{\varepsilon(1-\varepsilon)[\gamma(1-\varepsilon)-\varepsilon]}\frac{dq}{dR_2} \end{gathered} \tag{14}$$

If the dimensionless radius $R_2(q)$ of the detonation-wave front is not known, then by replacing in the equation of conservation of energy (8) the functions $f(\lambda, q)$, $g(\lambda, q)$ and $h(\lambda, q)$ with the corresponding expressions from the solution (13) in which the functions H_1, H_2, H_3 and H_4 are defined by (14), we obtain for $R_2(q)$ the following second order ordinary differential equation:

$$\begin{gathered} A_1R_2\frac{d^2q}{dR_2^2} + \left\{2A_1(\nu+1+\varepsilon-\omega) + \frac{R_2(1-\varepsilon)}{\nu q(\gamma-1)(\nu-\omega+2\varepsilon)} - \right. \\ \left.\nu(1-\gamma)(1-\varepsilon)A_1\right\}\frac{dq}{dR_2} + R_2A_1\left\{\frac{1}{[\gamma(1-\varepsilon)-\varepsilon]^2 q} - \frac{1}{q} + \right. \\ \left.\frac{4(\gamma-1)q}{[\gamma(1-\varepsilon)-\varepsilon](\nu-\omega+2\varepsilon)} - \frac{2\nu q(1-\gamma)^2}{[\gamma(1-\varepsilon)-\varepsilon](\nu-\omega+2\varepsilon)}\right\}\left(\frac{dq}{dR_2}\right)^2 - \\ \frac{1-\varepsilon}{(\nu-\omega+2\varepsilon)}\left\{\frac{\nu-\omega+\varepsilon}{\nu(\gamma-1)} + \frac{1-\varepsilon}{2}\right\} + \frac{q^2}{\gamma\sigma_\nu R_2^{\nu-\omega}} + \frac{q^2}{\nu-\omega} = 0 \end{gathered} \tag{15}$$

where

$$A_1 = \frac{2R_2q}{\nu[\gamma(1-\varepsilon)-\varepsilon](\nu-\omega+2\varepsilon)^2}$$

Taking the parameter q as the independent variable and $R_2(q)$ as well as $\sigma(q) = dq / dR_2$ as the functions to be determined, we obtain a system of two ordinary equations which we integrated numerically for various values of γ. The values of $R_2(q)$ obtained were then used to determine the dimensionless characteristics, that is the pressure, velocity and density, from the formulas (13).

Figure 1 depicts, for several values of the dimensionless parameter q, the distributions

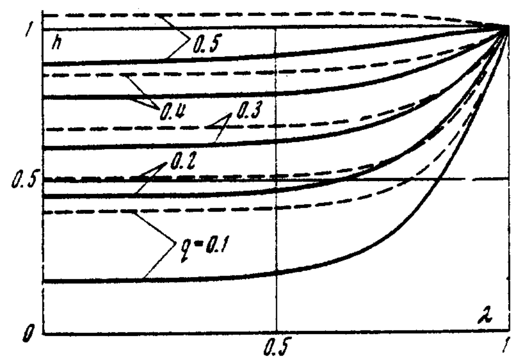

Fig. 1

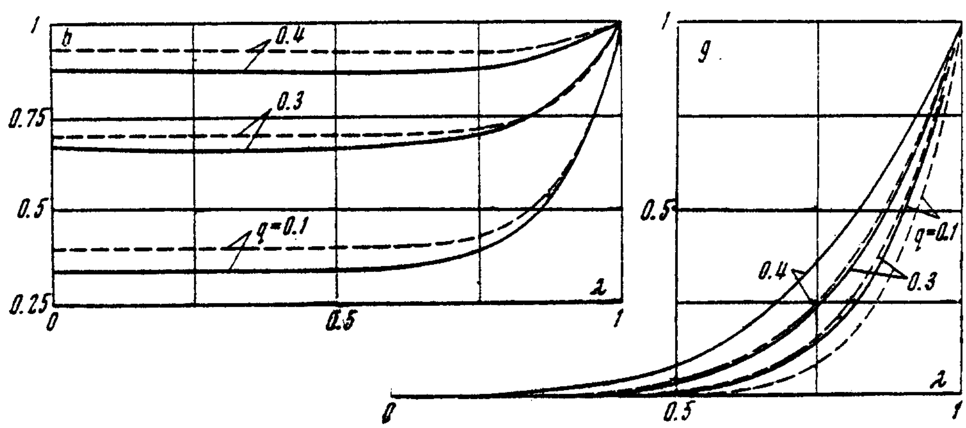

Fig. 2

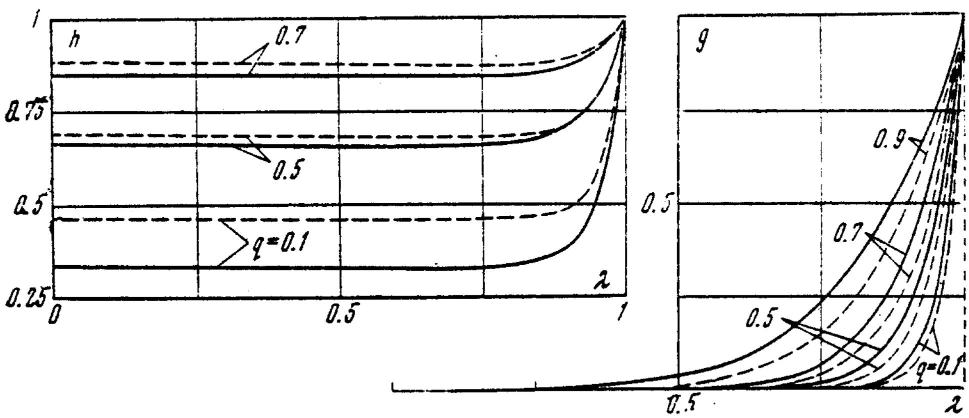

Fig. 3